Int. J. Heat Mass Transfer. Vol. 31, No. 8, pp. 1645–1653, 1988
Printed in Great Britain

0017-9310/88 \$3.00+0.00

Analytical study of vacuum-sublimation in an initially partially filled frozen porous medium with recondensation

Y. C. FEY and M. A. BOLES
Department of Mechanical and Aerospace Engineering, North Carolina State University, Raleigh, NC 27695-7910, U.S.A.

(*Received* 17 *April* 1987 *and in final form* 22 *January* 1988)

Abstract—Analytical solutions are developed for vacuum-sublimation in an initially partially filled, frozen, semi-infinite porous medium. Since the porous medium is partially filled, the vapor generated at the sublimation interface will convect into both the dried and frozen regions. The mass transfer of vapor through the porous medium is modeled by the application of Darcy's and Fick's laws. Recondensation of vapor occurs when the vapor convects into the frozen region. The sublimation interface temperature, pressure and vapor concentration are obtained in addition to the interface position. Moreover, the effects of vapor recondensation and the significant system parameters on the non-dimensional interface position are studied. The results show the increased sublimation rate when the vapor recondenses in the frozen region.

INTRODUCTION

THE APPLICATION of the vacuum-sublimation drying process has been widely used in food, medical and chemical industries over the past decades. The advantages of this dehydration process are that the shape and quality of the heat-sensitive products can be maintained. For vacuum-sublimation of the initially fully filled frozen porous medium, no moisture movement exists in the frozen region, there is no need to include the transport equations for vapor concentration and pressure fields in this region. In this work, we study the case for which the semi-infinite frozen porous medium is initially partially filled with the bound substance. Since the voids are partially filled, a concentration of vapor may initially exist in the frozen voids. When vacuum-sublimation begins, the vapor concentration and pressure have the highest values at the interface. Therefore, the vapor will convect into both the dried and frozen regions according to Darcy's and Fick's laws [1–3]. Since the temperature in the dried region is higher than the interface temperature, the vapor from the sublimation interface is superheated and no recondensation occurs in this region. However, the temperature in the frozen region is lower than the interface temperature. When the vapor generated at the interface flows into the frozen region, vapor recondensation will occur. As the vapor recondenses, three simultaneous interactions exist, namely: (a) the latent heat of condensation will be released which will affect the energy equation like a source term; (b) the amount of recondensation of vapor will affect the mass conservation equation like a sink term; (c) the vapor pressure will be lowered due to recondensation of vapor which will affect the equation of pressure field like a sink term.

Based upon the work of refs. [2, 4, 5], this paper proposes a model which takes into account the transport equations for the vapor concentration and pressure fields in the frozen region as well as the dried region. Also, the effect of vapor recondensation is included in the transport equations of the frozen region. The present formulation leads to an exact analytical solution for the transient model of the vacuum-sublimation process in the initially partially filled frozen porous media. It is of prime interest to know the effects of vapor recondensation and the important new parameters associated with vacuum-sublimation in a partially filled porous medium on the sublimation interface position. This study, therefore, presents the figures to show the physical trends involved in the vacuum-sublimation process in the initially partially filled frozen porous media.

STATEMENT OF THE PROBLEM

An initially partially filled frozen porous, semi-infinite medium is exposed to an environment where the pressure and vapor concentration are maintained below the triple point of the bound substance, and the temperature is higher than the initial temperature of the medium. The amount of residual non-condensable gases such as air in the partially filled voids is very small as compared to that of vapor. The porous medium is assumed to be composed of very small solid particles of the same size. Also, the medium is isotropic, homogeneous and rigid. Figure 1 illustrates an initially partially filled, semi-infinite frozen porous medium where the temperature, pressure, frozen moisture content and saturated vapor concentration are initially constant throughout the medium. At time greater than zero the vacuum-sublimation process

NOMENCLATURE

c_p	specific heat
C	molar concentration of vapor
C_0	molar concentration of frozen bound substance
C_{0v}	initial saturated molar concentration of vapor
$\bar{C}$	non-dimensional molar concentration of vapor, C/C_3
$\bar{C}_0$	non-dimensional molar concentration of frozen bound substance, C_0/C_3
K	effective thermal conductivity
K_{21}	thermal conductivity ratio, K_2/K_1
l	latent heat of sublimation
L	non-dimensional latent heat of sublimation, $l/(R_0T_3)$
Lu_i	Luikov moisture diffusivity in region i, $i = 1$ or 2, α_{mi}/α_2
Lu_{pi}	Luikov filtration diffusivity in region i, $i = 1$ or 2, α_{pi}/α_2
M_m	molecular weight of bound substance
P	pressure
P_0	initial pressure in the partially filled porous medium
$\bar{P}$	non-dimensional pressure, P/P_3
Q	$C_3M_m\alpha_2l/(T_3K_1)$
r	recondensation factor
R	non-dimensional gas constant, $C_3R_0T_3/P_3$
R_0	universal gas constant
$s(t)$	position of sublimation interface
t	time
T	temperature
T_0	initial temperature
$U_1(\eta)$	transformation variable, $T_1(\eta)+\delta_1C_1(\eta)+\delta_2P_1(\eta)$
V_1	δ_1C_3/T_3
V_2	δ_2P_3/T_3
x	space coordinate
$Z_1(\eta)$	transformation variable, $P_1(\eta)+\beta_1C_1(\eta)$
$Z_2(\eta)$	transformation variable, $P_2(\eta)+\beta_2C_2(\eta)$.

Greek symbols

α	effective thermal diffusivity
α_{mi}	effective moisture diffusivity in region i, $i = 1$ or 2
α_{pi}	filtration motion diffusivity coefficient of vapor in region i, $i = 1$ or 2
α_{21}	thermal diffusivity ratio, α_2/α_1
β_1	$[\alpha_{m1}-(\varepsilon-\omega)\alpha_{p1}]/\kappa_1$
β_2	$(\alpha_{m2}-\varepsilon\alpha_{p2})/\kappa_2$
δ_1	transformation coefficient, equation (24)
δ_2	transformation coefficient, equation (25)
Δ_i	non-dimensional permeability in region i, $i = 1$ or 2, $-P_3/(\beta_iC_3)$
ε	porosity
η	similarity variable, $x/(2\sqrt{(\alpha_2t)})$
θ	non-dimensional temperature, T/T_3
θ_0	non-dimensional initial temperature, T_0/T_3
κ_i	permeability in region i, $i = 1$ or 2
λ	non-dimensional position of sublimation interface, $s(t)/(2\sqrt{(\alpha_2t)})$
ρ	density
ω	volume fraction of frozen bound substance.

Subscripts

1	frozen region, $s(t) < x < \infty$
2	dried region, $0 < x < s(t)$
3	at triple point of bound substance
s	at surface, $x = 0$
v	at sublimation interface, $x = s(t)$.

begins, and the boundary conditions at the surface, $x = 0$, are maintained such that the temperature is above the initial temperature but below the scorch temperature of the porous medium. Also, the vapor concentration and vapor pressure are below the triple point values. The moving sublimation interface located by $x = s(t)$ is assumed sharply thin and separates the porous medium into the dried and frozen regions. The temperature, vapor concentration and pressure at the sublimation interface are in equilibrium and unknown but are assumed to have constant values which will be determined during the solution. The frozen region is assumed to maintain its initial uniform frozen moisture content throughout the process due to the very small amount of vapor recondensation as compared to the frozen moisture content; and the vapor movements exist in both the frozen and dried regions as a result of the interactions of temperature, vapor concentration and pressure gradients. In this work, we propose a theoretical model which applies the heat and mass transport equations to both the dried region and frozen region. Furthermore, the recondensation terms are included in the transport equations of the frozen region. To formulate the theoretical model of the above-mentioned vacuum-sublimation process, the additional assumptions are made as follows:

(1) the one-dimensional heat and mass transfer is considered;

(2) the heat radiation, heat convection, thermal expansion of the medium, Soret and Dufour effects are assumed small and negligible;

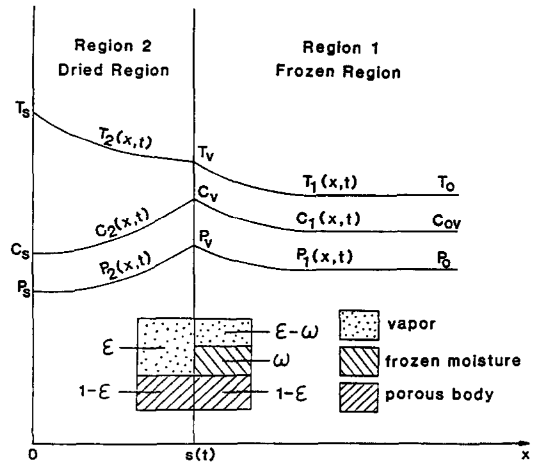

FIG. 1. Schematic description for the vacuum-sublimation problem in an initially partially filled frozen porous medium.

(3) the thermofluid properties remain constant but may be different for different regions;

(4) the Clapeyron equation and ideal gas law are assumed valid at the sublimation interface.

According to the above assumptions, the vacuum-sublimation process in an initially partially filled frozen porous medium may be formulated by the following equations:

$$\frac{\partial T_1(x,t)}{\partial t} = \alpha_1 \frac{\partial^2 T_1(x,t)}{\partial x^2} + \frac{rl(\varepsilon-\omega)}{(\rho c_p)_1}\frac{\partial C_1(x,t)}{\partial t}, \quad s(t) < x < \infty \quad (1)$$

$$\frac{\partial T_2(x,t)}{\partial t} = \alpha_2 \frac{\partial^2 T_2(x,t)}{\partial x^2}, \quad 0 < x < s(t) \quad (2)$$

$$(\varepsilon-\omega)\frac{\partial C_1(x,t)}{\partial t} = \alpha_{m1}\frac{\partial^2 C_1(x,t)}{\partial x^2} + \kappa_1 \frac{\partial^2 P_1(x,t)}{\partial x^2} - r(\varepsilon-\omega)\frac{\partial C_1(x,t)}{\partial t}, \quad s(t) < x < \infty \quad (3)$$

$$\varepsilon\frac{\partial C_2(x,t)}{\partial t} = \alpha_{m2}\frac{\partial^2 C_2(x,t)}{\partial x^2} + \kappa_2 \frac{\partial^2 P_2(x,t)}{\partial x^2}, \quad 0 < x < s(t) \quad (4)$$

$$\frac{\partial P_1(x,t)}{\partial t} = \alpha_{p1}\frac{\partial^2 P_1(x,t)}{\partial x^2} - r\frac{\partial P_1(x,t)}{\partial t}, \quad s(t) < x < \infty \quad (5)$$

$$\frac{\partial P_2(x,t)}{\partial t} = \alpha_{p2}\frac{\partial^2 P_2(x,t)}{\partial x^2}, \quad 0 < x < s(t) \quad (6)$$

where equations (1) and (2) describe the temperature distribution in the frozen and dried regions, respectively. Equations (3)–(6), based upon the Luikov system [2, 3] which is also employed in the previous work [4, 6], describe the vapor concentration and pressure fields in the frozen region and dried region, respectively. It is noted that the first and second terms on the right-hand side of equations (3) and (4) represent mass transfer of vapor due to Fick's and Darcy's flows, respectively. We also note that the terms with recondensation factor r on the right-hand side of equations (1), (3) and (5) are the source term of heat and the sink terms of vapor concentration and pressure, respectively, due to the recondensation of vapor from the sublimation interface in the frozen region.

The boundary and initial conditions are

$$T_1(x,0) = T_1(\infty,t) = T_0 \quad (7)$$

$$C_1(x,0) = C_1(\infty,t) = C_{0v} \quad (8)$$

$$P_1(x,0) = P_1(\infty,t) = P_0 \quad (9)$$

$$T_2(0,t) = T_s \quad (10)$$

$$C_2(0,t) = C_s \quad (11)$$

$$P_2(0,t) = P_s. \quad (12)$$

At the sublimation interface the conditions are

$$T_1(s,t) = T_2(s,t) = T_v \quad (13)$$

$$C_1(s,t) = C_2(s,t) = C_v \quad (14)$$

$$P_1(s,t) = P_2(s,t) = P_v \quad (15)$$

where T_v, C_v and P_v are the interface temperature, molar concentration and pressure of vapor, respectively, which are the unknown constants.

The Clapeyron equation relating the latent heat of sublimation to the interface conditions, from equation (8) in ref. [6], is

$$\frac{C_v T_v}{C_3 T_3} = \exp\left[\frac{l}{R_0}\left(\frac{1}{T_3} - \frac{1}{T_v}\right)\right]. \quad (16)$$

By applying the ideal gas law at the sublimation interface, we have

$$P_v = C_v R_0 T_v. \quad (17)$$

The energy and moisture balances at the interface are

$$-K_2\frac{\partial T_2(s,t)}{\partial x} + K_1\frac{\partial T_1(s,t)}{\partial x} = \omega C_0 M_m l \frac{\mathrm{d}s(t)}{\mathrm{d}t} \quad (18)$$

$$-\alpha_{m1}\frac{\partial C_1(s,t)}{\partial x} - \kappa_1\frac{\partial P_1(s,t)}{\partial x} + \alpha_{m2}\frac{\partial C_2(s,t)}{\partial x} + \kappa_2\frac{\partial P_2(s,t)}{\partial x} = \omega(C_0 - C_v)\frac{\mathrm{d}s(t)}{\mathrm{d}t}. \quad (19)$$

SOLUTION OF THE PROBLEM

The system of differential equations (1)–(6) cannot be solved directly due to the simultaneities and couplings among them. Following the similar procedure in ref. [4], we introduce the non-dimensional similarity variable

$$\eta = \frac{x}{2\sqrt{(\alpha_2 t)}} \quad (20)$$

into equations (1)–(19). Equations (1), (3) and (5) are coupled due to the recondensation terms and the vapor concentration and pressure gradient terms; while equations (4) and (6) are coupled due to the pressure gradient terms. Hence, there is a need to define three new variables $U_1(\eta)$, $Z_1(\eta)$ and $Z_2(\eta)$ as in equations (21)–(23) to decouple the foregoing coupled equations, respectively

$$U_1(\eta) = T_1(\eta)+\delta_1 C_1(\eta)+\delta_2 P_1(\eta) \quad (21)$$

$$Z_1(\eta) = P_1(\eta)+\beta_1 C_1(\eta) \quad (22)$$

$$Z_2(\eta) = P_2(\eta)+\beta_2 C_2(\eta) \quad (23)$$

where

$$\delta_1 = \frac{(\varepsilon-\omega)rl\alpha_{m1}}{[(1+r)(\varepsilon-\omega)\alpha_1-\alpha_{m1}](\rho c_p)_1} \quad (24)$$

$$\delta_2 = \frac{(1+r)(\varepsilon-\omega)rl\kappa_1\alpha_1}{[(1+r)\alpha_1-\alpha_{p1}][(1+r)(\varepsilon-\omega)\alpha_1-\alpha_{m1}](\rho c_p)_1} \quad (25)$$

and

$$\beta_1 = \frac{\alpha_{m1}-(\varepsilon-\omega)\alpha_{p1}}{\kappa_1} \quad (26)$$

$$\beta_2 = \frac{\alpha_{m2}-\varepsilon\alpha_{p2}}{\kappa_2}. \quad (27)$$

The location of the interface is assumed to be given by

$$s(t) = 2\lambda\sqrt{(\alpha_2 t)} \quad (28)$$

where λ is an unknown constant to be determined during the solution.

With the introduction of the new variables η and λ, we note that the dried region corresponds to $0<\eta<\lambda$, and the frozen region corresponds to $\lambda<\eta<\infty$. The problem is transformed to the following system of ordinary differential equations, equations (29)–(34), with variable coefficients subject to the transformed boundary and interface conditions:

$$\frac{d^2U_1(\eta)}{d\eta^2}+2\frac{\alpha_2}{\alpha_1}\eta\frac{dU_1(\eta)}{d\eta} = 0, \quad \lambda<\eta<\infty \quad (29)$$

$$\frac{d^2T_2(\eta)}{d\eta^2}+2\eta\frac{dT_2(\eta)}{d\eta} = 0, \quad 0<\eta<\lambda \quad (30)$$

$$\frac{d^2P_1(\eta)}{d\eta^2}+2(1+r)\frac{\alpha_2}{\alpha_{p1}}\eta\frac{dP_1(\eta)}{d\eta} = 0, \quad \lambda<\eta<\infty \quad (31)$$

$$\frac{d^2P_2(\eta)}{d\eta^2}+2\frac{\alpha_2}{\alpha_{p2}}\eta\frac{dP_2(\eta)}{d\eta} = 0, \quad 0<\eta<\lambda \quad (32)$$

$$\frac{d^2Z_1(\eta)}{d\eta^2}+2(1+r)(\varepsilon-\omega)\frac{\alpha_2}{\alpha_{m1}}\eta\frac{dZ_1(\eta)}{d\eta} = 0, \quad \lambda<\eta<\infty \quad (33)$$

$$\frac{d^2Z_2(\eta)}{d\eta^2}+2\varepsilon\frac{\alpha_2}{\alpha_{m2}}\eta\frac{dZ_2(\eta)}{d\eta} = 0, \quad 0<\eta<\lambda. \quad (34)$$

The system of equations (29)–(34) can be solved exactly for U_1, T_2, P_1, P_2, Z_1 and Z_2 with the transformed boundary and interface conditions, and the solutions for T_1, C_1 and C_2 are then obtained from equations (21)–(23). After substituting the above solutions into the interface equations and performing the required manipulations, we obtained the four transcendental interface equations. By using the non-dimensional parameters defined in the nomenclature, the solutions for the vacuum-sublimation model are presented as follows:

$$\theta_1(\eta) = \theta_0+[(\theta_v-\theta_0)+V_1(\bar{C}_v-\bar{C}_{0v})+V_2(\bar{P}_v-\bar{P}_0)] \times\frac{\operatorname{erfc}(\sqrt{(\alpha_{21})}\eta)}{\operatorname{erfc}(\sqrt{(\alpha_{21})}\lambda)}+V_1\left\{[\Delta_1(\bar{P}_v-\bar{P}_0)-(\bar{C}_v-\bar{C}_{0v})] \times\frac{\operatorname{erfc}\left(\sqrt{\left(\frac{(1+r)(\varepsilon-\omega)}{Lu_1}\right)}\eta\right)}{\operatorname{erfc}\left(\sqrt{\left(\frac{(1+r)(\varepsilon-\omega)}{Lu_1}\right)}\lambda\right)}+\Delta_1(\bar{P}_0-\bar{P}_v) \times\frac{\operatorname{erfc}\left(\sqrt{\left(\frac{1+r}{Lu_{p1}}\right)}\eta\right)}{\operatorname{erfc}\left(\sqrt{\left(\frac{1+r}{Lu_{p1}}\right)}\lambda\right)}\right\}-V_2(\bar{P}_v-\bar{P}_0) \times\frac{\operatorname{erfc}\left(\sqrt{\left(\frac{1+r}{Lu_{p1}}\right)}\eta\right)}{\operatorname{erfc}\left(\sqrt{\left(\frac{1+r}{Lu_{p1}}\right)}\lambda\right)} \quad (35)$$

$$\theta_2(\eta) = \theta_s+(\theta_v-\theta_s)\frac{\operatorname{erf}(\eta)}{\operatorname{erf}(\lambda)} \quad (36)$$

$$\bar{P}_1(\eta) = \bar{P}_0+(\bar{P}_v-\bar{P}_0)\frac{\operatorname{erfc}\left(\sqrt{\left(\frac{1+r}{Lu_{p1}}\right)}\eta\right)}{\operatorname{erfc}\left(\sqrt{\left(\frac{1+r}{Lu_{p1}}\right)}\lambda\right)} \quad (37)$$

$$\bar{P}_2(\eta) = \bar{P}_s+(\bar{P}_v-\bar{P}_s)\frac{\operatorname{erf}(\eta/\sqrt{(Lu_{p2})})}{\operatorname{erf}(\lambda/\sqrt{(Lu_{p2})})} \quad (38)$$

$$\bar{C}_1(\eta) = \bar{C}_{0v}+[\Delta_1(\bar{P}_0-\bar{P}_v)+(\bar{C}_v-\bar{C}_{0v})] \times\frac{\operatorname{erfc}\left(\sqrt{\left(\frac{(1+r)(\varepsilon-\omega)}{Lu_1}\right)}\eta\right)}{\operatorname{erfc}\left(\sqrt{\left(\frac{(1+r)(\varepsilon-\omega)}{Lu_1}\right)}\lambda\right)}+\Delta_1(\bar{P}_v-\bar{P}_0) \times\frac{\operatorname{erfc}\left(\sqrt{\left(\frac{1+r}{Lu_{p1}}\right)}\eta\right)}{\operatorname{erfc}\left(\sqrt{\left(\frac{1+r}{Lu_{p1}}\right)}\lambda\right)} \quad (39)$$

$$\bar{C}_2(\eta) = \bar{C}_s + [\Delta_2(\bar{P}_s - \bar{P}_v) + (\bar{C}_v - \bar{C}_s)] \times \frac{\operatorname{erf}\left(\sqrt{\left(\frac{\varepsilon}{Lu_2}\right)}\eta\right)}{\operatorname{erf}\left(\sqrt{\left(\frac{\varepsilon}{Lu_2}\right)}\lambda\right)} + \Delta_2(\bar{P}_v - \bar{P}_s) \times \frac{\operatorname{erf}\left(\frac{\eta}{\sqrt{(Lu_{p2})}}\right)}{\operatorname{erf}\left(\frac{\lambda}{\sqrt{(Lu_{p2})}}\right)}. \quad (40)$$

The energy and moisture balances at the sublimation interface become

$$\frac{K_{21}(\theta_s - \theta_v)\,\mathrm{e}^{-\lambda^2}}{\operatorname{erf}(\lambda)} - \frac{\sqrt{(\alpha_{21})}[(\theta_v - \theta_0) + V_1(\bar{C}_v - \bar{C}_{0v}) + V_2(\bar{P}_v - \bar{P}_0)]\,\mathrm{e}^{-\alpha_{21}\lambda^2}}{\operatorname{erfc}(\sqrt{(\alpha_{21})}\lambda)} + \frac{V_1[\Delta_1(\bar{P}_0 - \bar{P}_v) + (\bar{C}_v - \bar{C}_{0v})] \times \sqrt{((1+r)(\varepsilon-\omega))}\,\mathrm{e}^{-((1+r)(\varepsilon-\omega)\lambda^2)/Lu_1}}{\sqrt{(Lu_1)}\operatorname{erfc}\left(\sqrt{\left(\frac{(1+r)(\varepsilon-\omega)}{Lu_1}\right)}\lambda\right)} + \frac{(V_1\Delta_1 + V_2)(\bar{P}_v - \bar{P}_0)\sqrt{(1+r)}\,\mathrm{e}^{-((1+r)\lambda^2)/Lu_{p1}}}{\sqrt{(Lu_{p1})}\operatorname{erfc}\left(\sqrt{\left(\frac{1+r}{Lu_{p1}}\right)}\lambda\right)} = \sqrt{(\pi)}\omega\bar{C}_0 Q\lambda \quad (41)$$

and

$$\frac{\sqrt{((1+r)(\varepsilon-\omega)Lu_1)}[\Delta_1(\bar{P}_0 - \bar{P}_v) + (\bar{C}_v - \bar{C}_{0v})]\,\mathrm{e}^{-((1+r)(\varepsilon-\omega)\lambda^2)/Lu_1}}{\operatorname{erfc}\left(\sqrt{\left(\frac{(1+r)(\varepsilon-\omega)}{Lu_1}\right)}\lambda\right)} + \frac{\sqrt{(1+r)(\varepsilon-\omega)}\sqrt{(Lu_{p1})}\Delta_1(\bar{P}_v - \bar{P}_0)\,\mathrm{e}^{-((1+r)\lambda^2)/Lu_{p1}}}{\operatorname{erfc}\left(\sqrt{\left(\frac{1+r}{Lu_{p1}}\right)}\lambda\right)} + \frac{\sqrt{(\varepsilon Lu_2)}[\Delta_2(\bar{P}_s - \bar{P}_v) + (\bar{C}_v - \bar{C}_s)]\,\mathrm{e}^{-\varepsilon\lambda^2/Lu_2}}{\operatorname{erf}\left(\sqrt{\left(\frac{\varepsilon}{Lu_2}\right)}\lambda\right)} + \frac{\varepsilon\sqrt{(Lu_{p2})}\Delta_2(\bar{P}_v - \bar{P}_s)\,\mathrm{e}^{-\lambda^2/Lu_{p2}}}{\operatorname{erf}\left(\frac{\lambda}{\sqrt{(Lu_{p2})}}\right)} = \sqrt{\pi}\lambda\omega(\bar{C}_0 - \bar{C}_v). \quad (42)$$

The Clapeyron equation becomes

$$\bar{C}_v\theta_v = \exp[L(1 - 1/\theta_v)] \quad (43)$$

and the ideal gas law becomes

$$\bar{P}_v = \bar{C}_v R\theta_v. \quad (44)$$

The non-dimensional sublimation interface position λ and non-dimensional temperature θ_v, pressure $\bar{P}_v$, molar concentration $\bar{C}_v$ at the interface are then obtained by numerically solving the simultaneous equations, equations (41)–(44). Once the interface constants are known, equations (35)–(40) readily yield the exact solution to the vacuum-sublimation problem of an initially partially filled frozen semi-infinite porous medium.

RESULTS AND DISCUSSION

To better understand the effects of vapor recondensation and significant new parameters in the frozen region on the vacuum-sublimation process in an initially partially filled frozen porous medium, sample calculations are performed for illustration. Since the sublimation rate is of major interest and is proportional to λ during the sublimation drying process, results are now presented which illustrate the effects of vapor recondensation and some important parameters on the non-dimensional interface position. On the figures presented in this study, only the parameters having values different from the reference values are indicated. The selected reference values are: $\omega = 0.1$, $\varepsilon = 0.8$, $\Delta_1 = 4.0$, $\Delta_2 = 4.2$, $Lu_1 = 1.0$, $Lu_2 = 1.0$, $Lu_{p1} = 300$, $Lu_{p2} = 300$, $R = 1.0$, $\alpha_{21} = 0.8$, $K_{21} = 0.8$, $\theta_0 = 0.96$, $\bar{P}_0 = 0.25$, $\bar{C}_{0v} = 0.25$, $\bar{C}_0 = 1000$, $L = 1$, $Q = 0.1 \times 10^{-6}$, $V_1 = 0.6 \times 10^{-6}$, $V_2 = -0.4 \times 10^{-5}$, $\theta_s = 1.0$, $\bar{P}_s = 0.25$, $\bar{C}_s = 0.2$.

The results of this study show that a faster sublimation rate is predicted with increasing recondensation factor r. This result demonstrates that the recondensation of vapor in the frozen region has a favorable effect on the vacuum-sublimation process in a partially filled porous medium. The sublimation rate depends upon the removal of vapor from the sublimation interface. As the recondensation factor increases, the pressure gradient in the frozen region at the sublimation interface increases. This pressure gradient results in a larger vapor flow from the sublimation interface into the frozen region as governed by Darcy's law. In the following figures, the recondensation factor r is chosen at 0, 0.4 and 0.8. The zero value of r corresponds to the case of no recondensation of vapor. It should be noted that a portion of the vapor generated by the sublimation process may flow into the frozen region and recondense; thus, r is a measure of the fraction of the rate of vapor concentration increase in the frozen region that does recondense. The enhancements in λ are defined as $[(\lambda_{r=0.8} - \lambda_{r=0})/\lambda_{r=0}] \cdot 100$.

Figures 2 and 3 show the effects of recondensation factor r, non-dimensional permeability Δ_1 and Luikov filtration diffusivity Lu_{p1} on non-dimensional interface position λ. A comparison between cases $r = 0.8$ and 0 indicates an enhancement in λ from 0.59% at $\Delta_1 = 1.0$ rapidly to 2.72% at $\Delta_1 = 7.0$ and from 0.96% at $Lu_{p1} = 100$ to 2.42% at $Lu_{p1} = 600$. As the permeability Δ_1 and Luikov filtration diffusivity Lu_{p1} increase, more vapor generated at the interface will

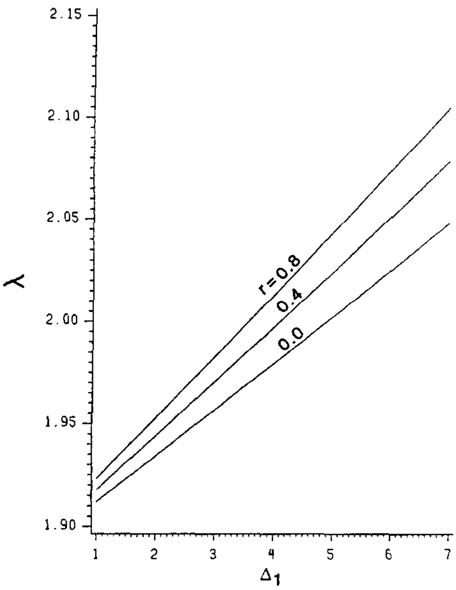

FIG. 2. Effects of recondensation factor and non-dimensional permeability Δ_1 on non-dimensional interface position.

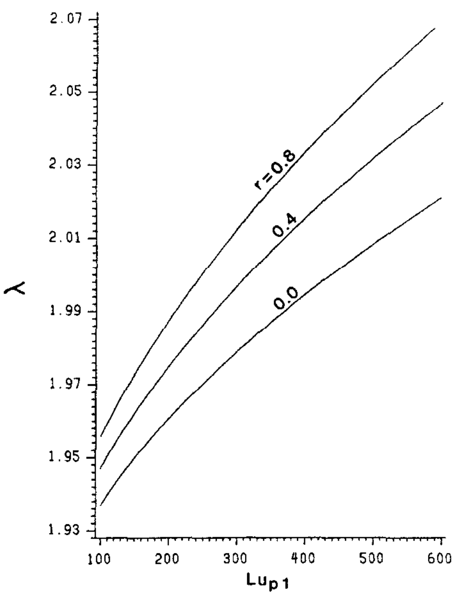

FIG. 3. Effects of recondensation factor and Luikov filtration diffusivity Lu_{p1} on non-dimensional interface position.

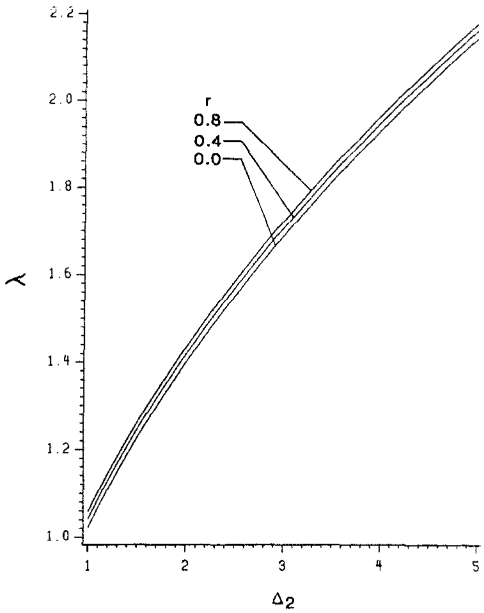

FIG. 4. Effects of recondensation factor and non-dimensional permeability Δ_2 on non-dimensional interface position.

convect into the frozen region due to less resistance to the vapor flow and faster propagation speed of pressure waves through the media. Thus, the amount of vapor recondensation increases which results in a higher pressure drop in the frozen region. Both the non-dimensional interface position and the significance of the effect of recondensation increase by this enhanced Darcy's flow.

Figures 4 and 5 illustrate the effects of recondensation factor r, non-dimensional permeability Δ_2 and Luikov filtration diffusivity Lu_{p2} on non-dimensional interface position λ. As compared to Figs. 2 and 3, the Δ_2 and Lu_{p2} have similar and greater effects on λ over the same ranges of non-dimensional permeability and Luikov filtration diffusivity. Comparing the results for $r = 0.8$ and 0 shows that the variations of the percent enhancement of λ change from 3.47% at $\Delta_2 = 1$ to 1.55% at $\Delta_2 = 5$ and from 2.94 to 1.31% for $Lu_{p2} = 100$ to 500. It is noted that the general trend of the percent enhancements in λ in these figures is the reverse of that in Figs. 2 and 3 over the ranges of Lu_p and Δ studied. Since the higher Δ_2 and Lu_{p2} allow the vapor to move more easily through the dried region, most of the vapor produced at the interface is convected outwardly, and the amount of vapor convected into the frozen region is reduced resulting in the smaller amount of vapor recondensation. Thus, as the Δ_2 and Lu_{p2} increase, one would expect the higher values of λ, but the effect of recondensation of vapor in the frozen region is more important for lower values of Δ_2 and Lu_{p2}.

Figure 6 depicts the effects of recondensation factor r and non-dimensional initial pressure $\bar{P}_0$ on non-dimensional interface position λ. When the initial pressure increases, the pressure gradients in the frozen region decrease, so that less vapor is convected into this region due to the weakened Darcy's flow. This results in the lower values of λ and the reduced effect of vapor recondensation on λ as $\bar{P}_0$ increases. For instance, as the recondensation factor r varies from 0 to 0.8, the enhancement in λ is 2.05% at $\bar{P}_0 = 0.1$ and only 0.38% at $\bar{P}_0 = 0.8$.

Figure 7 indicates the effects of recondensation factor r and non-dimensional surface pressure $\bar{P}_s$ on non-dimensional interface position λ for $\bar{C}_s = 0.1$. This figure shows the lower sublimation rate for higher values of surface pressure $\bar{P}_s$. A comparison of cases with $r = 0$ and 0.8 demonstrates that the λ is increased by 1.53% at $\bar{P}_s = 0.1$ and by 2.76% at $\bar{P}_s = 0.7$. This

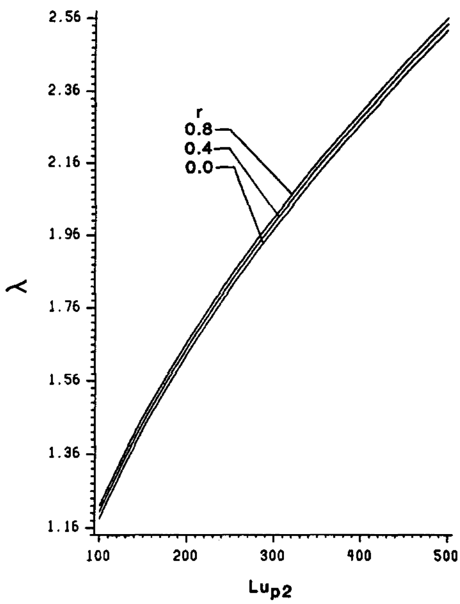

FIG. 5. Effects of recondensation factor and Luikov filtration diffusivity Lu_{p2} on non-dimensional interface position.

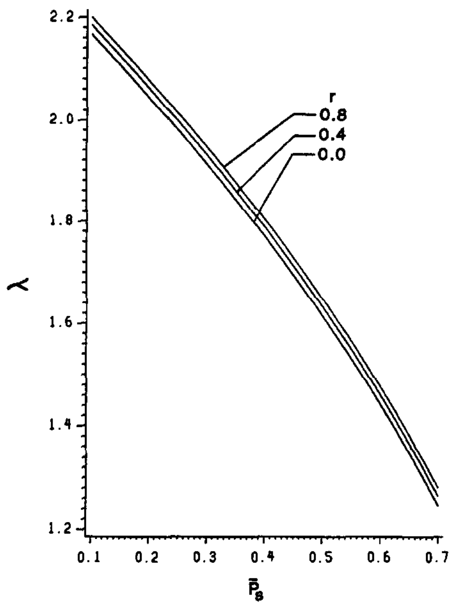

FIG. 7. Effects of recondensation factor and non-dimensional surface pressure on non-dimensional interface position.

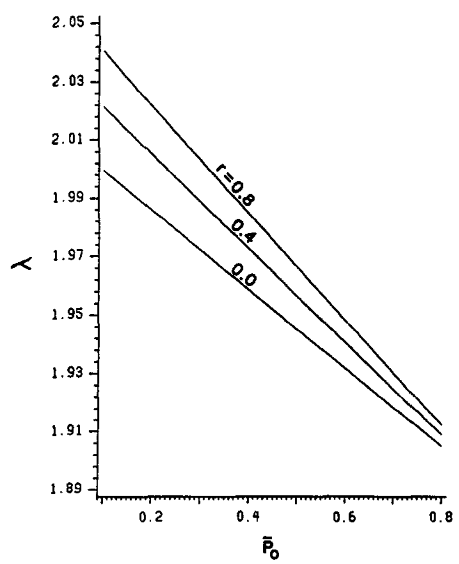

FIG. 6. Effects of recondensation factor and non-dimensional initial pressure on non-dimensional interface position.

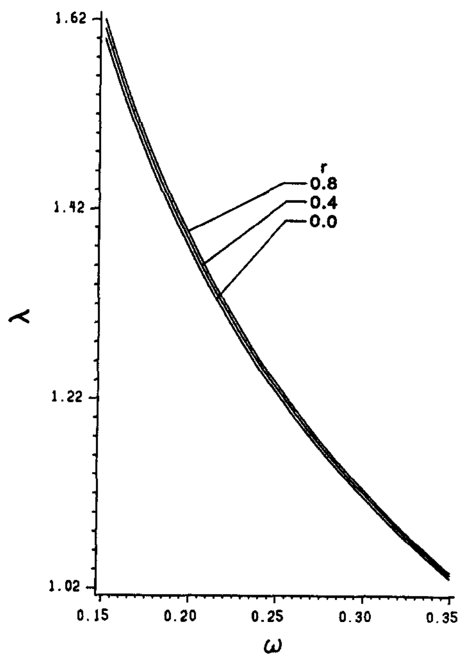

FIG. 8. Effects of recondensation factor and volume fraction of frozen bound substance on non-dimensional interface position.

is due to the fact that the higher value of $\bar{P}_s$ will hinder the vapor produced at the interface to be convected outwardly, and the amount of vapor convected into the frozen region is increased which causes the greater effect of vapor recondensation on the percent enhancement in λ. By comparison with Fig. 6, it is noted that $\bar{P}_s$ has a similar but greater effect on λ, but the physical trend of the percent enhancements of λ in this figure is the reverse of that in Fig. 6 over the ranges of non-dimensional pressure studied.

Figure 8 illustrates the effects of recondensation factor r and volume fraction of frozen bound substance ω on non-dimensional interface position λ. When ω increases, the non-dimensional interface position decreases significantly because there is more substance to be sublimated per unit volume at the interface. This results in the decreasing effect of the recondensation factor on λ for higher values of ω. As indicated in the figure, the enhancements in λ vary from 1.28% at $\omega = 0.15$ to only 0.57% at $\omega = 0.35$ between the cases of $r = 0.8$ and 0.

The important new characteristics for vacuum-sublimation in an initially partially filled frozen porous medium have been studied in the above. Results for other parameters such as L, $\bar{C}_0$, θ_s, θ_0, K_{21}, α_{21}, Lu_1, Lu_2, $\bar{C}_s$ and $\bar{C}_{0v}$ have also been obtained. The effects of variations in these parameters except $\bar{C}_{0v}$ on the non-dimensional interface position λ have been discussed in ref. [5]. However, the effect of $\bar{C}_{0v}$ is found to be small and is negligible. Therefore, the discussion concerning these parameters and the effects of vapor recondensation over the ranges of these parameters is omitted in this work.

CONCLUSION

According to the above discussion, the following conclusions for vacuum-sublimation in an initially partially filled frozen porous medium may be drawn.

(1) The new parameters (Δ_1, Lu_{p1}, $\bar{P}_0$, ω) for the frozen region associated with vacuum-sublimation in a partially filled porous medium have been shown to have definite effects on the sublimation rate, while the other new parameters ($\bar{C}_{0v}$, Lu_1) have been shown to have negligible effects. The effects of the corresponding dried region parameters (Δ_2, Lu_{p2}, $\bar{P}_s$) are more significant than those of the frozen region parameters (Δ_1, Lu_{p1}, $\bar{P}_0$). Also, the sublimation rate is closely dependent on ω.

(2) The vapor recondensation in the frozen region will increase the sublimation rate, but this effect is usually small and negligible. Moreover, the fluid-related parameters (Δ_1, Δ_2, Lu_{p1}, Lu_{p2}, $\bar{P}_0$, $\bar{P}_s$, ω, $\bar{C}_0$) and sublimation latent heat L influenced the general trend of variations of the percent enhancement in λ due to vapor recondensation more than did the thermo-related parameters (θ_s, θ_0, K_{21}, α_{21}).

(3) The general trend between λ and $\Delta\lambda$ (e.g. $\lambda_{r=0.8}-\lambda_{r=0}$) is that the (percent) enhancement in λ due to the effect of vapor recondensation increases with the higher values of λ over the ranges of most of the system parameters except for the parameters (Δ_2, Lu_{p2}, $\bar{P}_s$).

REFERENCES

1. J. C . Harper, Transport properties of gases in porous media at reduced pressures with reference to freeze-drying, *A.I.Ch.E. Jl* **8**, 298–302 (1962).
2. A. V. Luikov, *Heat and Mass Transfer in Capillary-porous Bodies*. Pergamon Press, Oxford (1966).
3. A. V. Luikov, Systems of differential equations of heat and mass transfer in capillary-porous bodies, *Int. J. Heat Mass Transfer* **18**, 1–14 (1975).
4. Y. C. Fey and M. A. Boles, An analytical study of the effect of Darcy's and Fick's laws on the sublimation of a frozen semi-infinite porous medium, *J. Heat Transfer* **109**, 1045–1048 (1987).
5. S. Lin, An exact solution of the sublimation problem in a porous medium, part II—with an unknown temperature and vapor concentration at the moving sublimation front, *J. Heat Transfer* **104**, 808–811 (1982).
6. Y. C. Fey and M. A. Boles, An analytical study of the effect of convection heat transfer on the sublimation of a frozen semi-infinite porous medium, *Int. J. Heat Mass Transfer* **30**, 771–779 (1987).
7. S. Lin, An exact solution of the sublimation problem in a porous medium, *J. Heat Transfer* **103**, 165–168 (1981).
8. K. Katayama and Y. Hayashi, Researches on vacuum freeze-drying, *Bull. J.S.M.E.* **15**, 344–353 (1972).
9. M. D. Mikhailov, Exact solution of temperature and moisture distributions in a porous half-space with moving evaporation front, *Int. J. Heat Mass Transfer* **18**, 797–804 (1975).
10. D. F. Dyer and J. E. Sunderland, Bulk and diffusional transport in the region between molecular and viscous flow, *Int. J. Heat Mass Transfer* **9**, 519–526 (1966).
11. D. F. Dyer and J. E. Sunderland, Heat and mass transfer mechanisms in sublimation dehydration, *J. Heat Transfer* 379–384 (Nov. 1968).
12. M. A. Boles and M. N. Ozisik, Exact solution for freezing in cylindrically symmetric, porous, moist media, *J. Heat Transfer* **105**, 401–403 (1983).
13. P. C. Carman, *Flow of Gases through Porous Media*. Butterworth, London (1956).
14. R. E. Sonntag and G. J. Van Wylen, *Introduction to Thermodynamics Classical and Statistical*, 2nd Edn. Wiley, New York (1982).

ETUDE ANALYTIQUE DE LA SUBLIMATION DANS LE VIDE ET RECONDENSATION POUR DES MILIEUX POREUX INITIALEMENT EMPLIS PARTIELLEMENT DE GLACE

Résumé—Des solutions analytiques sont développées pour la sublimation dans le vide dans un milieu poreux semi-infini partiellement empli et gelé. Puisque le milieu poreux est partiellement empli, la vapeur formée à l'interface de sublimation convecte à la fois dans les régions sèche et gelée. Le transfert de masse de la vapeur à travers le milieu poreux est modélisé par application des lois de Darcy et de Fick. La recondensation de la vapeur se produit quand la vapeur convecte dans la région gelée. La température à l'interface de sublimation, la pression et la concentration de vapeur sont obtenues en plus de la position de l'interface. On étudie les effets de la recondensation de la vapeur et des paramètres caractéristiques du système sur la position de l'interface. Les résultats montrent l'accroissement du taux de sublimation quand la vapeur se recondense dans la région gelée.

ANALYTISCHE UNTERSUCHUNG DER VAKUUM-SUBLIMATION IN EINEM ANFANGS TEILWEISE GEFÜLLTEN, GEFRORENEN PORÖSEN MEDIUM MIT REKONDENSATION

Zusammenfassung—Für die Vakuum-Sublimation in einem anfangs teilweise gefüllten, gefrorenen halbunendlichen porösen Medium werden analytische Lösungen entwickelt. Da das poröse Medium teilweise gefüllt ist, wird der an der Sublimationsgrenzfläche erzeugte Dampf sowohl in die getrockneten als auch in die gefrorenen Bereiche strömen. Der Dampftransport durch das poröse Medium wird durch Anwendung der Gesetze von Darcy und Fick dargestellt. Rekondensation des Dampfes tritt auf, wenn der Dampf in den gefrorenen Bereich strömt. Zusätzlich zur Position der Sublimationsgrenzfläche wird deren Temperatur, der Druck und die Dampfkonzentration berechnet. Darüber hinaus werden die Auswirkungen der Dampfrekondensation und der signifikanten Systemparameter auf die dimensionslose Position der Grenzfläche untersucht. Die Ergebnisse zeigen die wachsende Sublimationsgeschwindigkeit bei der Rekondensation von Dampf in den gefrorenen Bereichen.

АНАЛИТИЧЕСКОЕ ИССЛЕДОВАНИЕ ВАКУУМНОЙ СУБЛИМАЦИИ И РЕКОНДЕНСАЦИИ ЗАМЕРЗШЕЙ ЖИДКОСТИ, ВНАЧАЛЕ ЧАСТИЧНО ЗАПОЛНЯЮЩЕЙ ПОРИСТУЮ СРЕДУ

Аннотация—Получены аналитические решения для вакуумной сублимации замерзшей жидкости, вначале частично заполняющей полубесконечную пористую среду. Генерируемый на границе раздела фаз пар переносится конвекцией как в свободную часть пористой среды, так и в область, заполненную твердым сублиматом. Массоперенос пара через пористую среду моделируется с помощью законов Дарси и Фика. Реконденсация пара происходит при конвективном переносе в замороженную область. Определены температура, давление и концентрация пара на границе раздела фаз, а также местоположение последней. Кроме того, исследовано влияние реконденсации пара и основных параметров системы на положение границы раздела фаз, которое представлено в безразмерном виде. Результаты показывают, что в том случае, когда пар реконденсируется в области, заполненной замерзшей жидкостью, скорость сублимации возрастает.